An observation about the diagonal argument

The Diagonal Argument has the following form

- **If |N| = |R|, then there is a 1-to-1 function on N and onto R.**
- **There is no 1-to-1 function on N and onto R.**
- **It is not the case that |N| = |R|.**

SO!

- We are **not** proving that **|R| > |N|** by proving that there is some greater quantity of things – namely N + z.
- (That wouldn't work! N plus any finite number of things has the same cardinality as N. And even N plus |N|-many-things has the cardinality of N!)
- We are proving that **|N| ≠ |R|** by showing that there cannot be a 1-to-1 function on N and onto R: for any such possible function, Cantor will show you a contradiction.

Now we'll fill in

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  1. Suppose there were a 1-to-1 function on N and onto R. Call this f.
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  2. Then, using f, we could construct z.
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    1. Suppose there were a 1-to-1 function on N and onto R. Call this f.
    2. Then, using f, we could construct z.
    3. z is a real number (that is, $z \in R$).
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  2. Then, using f, we could construct z.
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  4. z is in the range of f (by 1 and 3).
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  3. z is a real number (that is, $z \in R$).
  4. z is in the range of f (by 1 and 3).
  5. z is not in the range of f (because for any $n \in N$, f(n) differs from z at least at the n^{th} decimal place).
- **It is not the case that |N| = |R|.**

Done!

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  4. z is in the range of f (by 1 and 3).
  5. z is not in the range of f (because for any $n \in N$, f(n) differs from z at least at the n^{th} decimal place).
  6. (4 and 5 contradict, so we conclude that the source of our error is line 1.)
- **It is not the case that |N| = |R|.**

Now we'll fill in the quick steps to >

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- Because N is a proper subset of R (N ⊂ R), there is a 1-to-1 function on N and into R (for example, the identity function). And so |N| ≤ |R|.

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- If it is not the case that |N| = |R| and it is the case that |N| ≤ |R|, then |R| > |N|.

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- |R| > |N|