Luitzen Brouwer

1862-1943

Principle founder of Intutionism

Intuitionist program

- Reject the Principle of the Excluded Middle. [The point here is to reject the following kind of reasoning: If I know that $\neg$**P** is false, I know that **P** is true.]
- We will say a formula is true only if we have a constructive proof of that formula. [A constructive proof is a proof that does not use *reductio ad absurdum* and relies upon intuitive steps and intuitive axioms.]

David Hilbert

1862-1943

Principle founder of Formalism

Formalist program

- Establish clearly the nature of logic, to serve as the foundation to our reasoning.
- For each mathematical theory T, find/create the basic concepts and the axioms of that theory.
- Clarify the nature of proofs in T, so that each proof in T is a finite list where each step is the product of syntactic rules of logic applied to our axioms of T.

[That last step is why this is called “formalism”—proofs are ideally reduced to application of syntactic rules.]

Hilbert's Problems

1900 Address:

The 23 Problems.

Including:

- Consistency
- Decidability
- Completeness

Hilbert’s Problems implicitly include the following questions

- Can we prove arithmetic is consistent?
- Can we prove arithmetic is complete?
- Can we prove arithmetic is decidable?

Hilbert’s Problems implicitly include the following questions

- A reasoning system is **consistent** if we cannot prove a falsehood.
- A reasoning system is **complete** if all the truths of the system are provable.
- A reasoning system is **decidable** if there is an effective procedure to determine if a formula is a theorem.