# Kant and Geometry

# Some examples of kinds of claims

- A priori
  - 2+2=4
  - Triangles have three sides.
- A posteriori
  - Lincoln was from Illinois.
  - Earth has one moon.
- Synthetic
  - Whales are mammals.
  - Nixon was president.
- Analytic
  - Bachelors are unmarried males.
  - Soaking things are wet.

# The synthetic *a priori*

- Synthetic *a priori* judgments are special because their possibility reveals something neither given in experience, but not derived simply from the analysis of other assumed contents.
- These reveal to us the things that agents bring to the construction of the (phenomenal) world.

# The Categories

- Of quantity
  - Unity
  - Plurality
  - Totality
- Of Quality
  - Reality
  - Negation
  - Limitation
- Of relation
  - Of inherence and subsistence
  - Of causality and dependence
  - Of community
- Of modality
  - Possibility
  - Existence
  - Necessity

-- From the Norman Kemp Smith Translation, 113 (B106)

# Also given in intuition and constituting a priori conditions on experience are:

- Space
- Time

# From *The Critique of Pure Reason*

Just as little is any fundamental proposition of pure geometry analytic. That the straight line between two points is the shortest, is a synthetic proposition. For my concept of *straight* contains nothing of quantity, but only of quality….Intuition, therefore, must be here called in; only by its aid is the synthesis possible.

-- Norman Kemp Smith Translation, 53 (B16)

# From *The Critique of Pure Reason*

Space is a necessary *a priori* representation, which underlies all outer intuitions…. It must therefore be regarded as the condition of the possibility of appearances, and not as a determination dependent upon them. It is an *a priori* representation, which necessarily underlies outer appearances…. For kindred reasons, geometrical propositions, that, for instance, in a triangle two sides together are greater than the third, can never be derived from the general concepts of line and triangle, but only from intuition, and this indeed *a priori,* with apodeictic certainty.

-- Norman Kemp Smith Translation, 68-69 (B39-40)

# From *The Critique of Pure Reason*

The apodeictic certainty of all geometrical propositions, and the possibility of their *a priori* construction, is grounded in this *a priori* necessity of space. Were this representation of space a concept acquired *a posteriori,* and derived from outer experience in general, the first principles of mathematical determination would be nothing but perceptions. They would therefore all share in the contingent character of perception; that there should be only one straight line between two points would not be necessary, but only what experience always teaches. What is derived from experience… is obtained through induction. We should therefore only be able to say that, so far as hitherto observed, no space has been found which has more than three dimensions.

-- Norman Kemp Smith Translation, 68-69 (A24)

# From *The Critique of Pure Reason*

Take, for instance, the proposition, "Two straight lines cannot enclose a space, and with them alone no figure is possible", and try to derive it from the concept of straight lines and of the number two. Or take the proposition, "Given three straight lines, a figure is possible", and try, in like manner, to derive it from the concepts involved. All your labor is vain; and you find that you are constrained to have recourse to intuition, as is always done in geometry…. If the object (the triangle) were something in itself, apart from any relation to you, the subject, how could you say that what necessarily exist in you as subjective conditions for the construction of a triangle, must of necessity belong to the triangle itself?

-- Norman Kemp Smith Translation, 68 (B39)

# What does Kant's view seem to entail?

- Space is a necessary precondition for experience, and our kind of being constitutes this space, space must be immutable for us
- The structure of space as given to us in intuition is geometry
- Euclid's geometry, including its statement that space has three dimensions, is the revealed nature of space

# Euclid's proto-Axiomatic method in *The Elements* (circa 300 BC)

- Some example definitions from *The Elements:*
  - 1: A point is that which has no part.
  - 2: A line is breadthless length.
  - 23: Parallel straight lines are straight lines that, being in the same plane and being extended indefinitely in both directions, do not meet one another in either direction.
- Some example postulates from *The Elements:*
  - 1: It is possible to draw a straight line from any point to any point.
  - 4: All right angles are equal to one another.
  - **5: If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if extended indefinitely, meet on that side on which are the angles less than the two right angles.**
- Euclid's parallel axiom from *The Elements*:
  - **Given a line and point, not more than one line can be drawn through the point such that the line is parallel to the given line.**

# Gauss, Bolyai, Lobachevsky

- Lobachevsky (1829), Gauss (circa 1824, in letters) and Bolyai (1832), discover that it appears we can deny the fifth postulate (and deny the parallel axiom) and develop a consistent geometry.
- In hyperbolic geometry, a line can have several parallels running through a single point not on the line.
- In spherical geometry, two straight lines crossing another line, both at 90 degrees, can still meet.

# Other odd features of non-Euclidean Geometries

- Hyperbolic: a triangle on a some spaces can have less than 180 degrees sum interior angles.
- Spherical: a triangle in a spherical geometry can have more than 180 degrees sum interior angles.
- Remember Kant's example of a necessary truth of geometry: "…in a triangle two sides together are greater than the third"? **This can fail to be true in a non-Euclidean Geometry!**

# What does this mean for Kant's view?

- If space is Euclidean (and until recently, everyone assumed it was), then why does our intuition, as given in our experience of space, (seem to) reveal that the results of non-Euclidean geometry are necessarily true?
- If space is not Euclidean, then why did it take us so long to intuit non-Euclidean geometry, given that space is the source of our intuitions about geometry?