Relations

A *binary relation* R on a set A is a set of ordered pairs of elements of A; R ⊆ A x A.

A binary relation R on a set A is called:

- *Reflexive* if: for every a∈A, <a, a> ∈ R;
- *Symmetric* if: for every a∈A and b∈A, <a,b>∈R only if <b,a>∈R;
- *Transitive* if: for every a∈A and b∈A and c∈A, if <a,b>∈R and <b,c>∈R then <a,c>∈R;
- *Antisymmetric* if: for every a∈A and b∈A, if <a,b>∈R and <b,a>∈R, then a=b.

A relation that is reflexive, symmetric, and transitive is called an *equivalence relation*.

A relation R on a set A that is reflexive, antisymmetric, and transitive, is called a (reflexive) *partial order*.

If a partial order R has no pairs <a,a>, for every a∈A, it is a *strict partial order*.

A partial order on a set A with the additional property that for every a∈A and b∈A, either a≤b or b ≤ a, is called a *total order*, or a *linear order*.

A total order on a set A is *well-ordered* if every non-empty subset of A has a least element.

Functions

A unary *function* from a set A and into a set B (these could be the same set) is a binary relation F such that for any a∈A and b∈B there is exactly one pair <a,b>∈F.

An n-ary *function* from a set A and into a set B (these could be the same set) is a n-ary relation F on A such that for every a_1∈A ... a_n∈A there is exactly one n+1-tuple <a_1 ... a_n, b> ∈ F.