

m5 in B&S4

1. $(\Diamond\Diamond\phi \rightarrow \Diamond\phi)$ theorem in S4
2. $\Box(\Diamond\Diamond\phi \rightarrow \Diamond\phi)$ nec, 1
3. $(\Box(\Diamond\Diamond\phi \rightarrow \Diamond\phi) \rightarrow (\Box\Diamond\Diamond\phi \rightarrow \Box\Diamond\phi))$ m2
4. $(\Box\Diamond\Diamond\phi \rightarrow \Box\Diamond\phi)$ MP 2, 3
5. $(\Diamond\phi \rightarrow \Box\Diamond\Diamond\phi)$ m3
6. $(\Diamond\phi \rightarrow \Box\Diamond\phi)$ chain rule 5, 4

m3 in S5

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| 1. $(\phi \rightarrow \Diamond\phi)$ | theorem in S5 |
| 2. $(\Diamond\phi \rightarrow \Box\Diamond\phi)$ | m5 |
| 3. $(\phi \rightarrow \Box\Diamond\phi)$ | chain rule 1, 2 |

m4 in S5

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|-----|--|--------------------------|
| 1. | $(\Diamond \neg \phi \rightarrow \Box \Diamond \neg \phi)$ | m5 |
| 2. | $(\neg \Box \Diamond \neg \phi \rightarrow \neg \Diamond \neg \phi)$ | contraposition 6 |
| 3. | $(\neg \Box \neg \Box \neg \neg \phi \rightarrow \neg \neg \Box \neg \neg \phi)$ | definitions, equivalence |
| 4. | $(\neg \Box \neg \Box \phi \rightarrow \Box \phi)$ | DN, DN, DN, 3 |
| 5. | $(\Diamond \Box \phi \rightarrow \Box \phi)$ | definition, 4 |
| 6. | $\Box (\Diamond \Box \phi \rightarrow \Box \phi)$ | nec |
| 7. | $(\Box (\Diamond \Box \phi \rightarrow \Box \phi) \rightarrow (\Box \Diamond \Box \phi \rightarrow \Box \Box \phi))$ | m2 |
| 8. | $(\Box \Diamond \Box \phi \rightarrow \Box \Box \phi)$ | mp 6, 7 |
| 9. | $(\Box \phi \rightarrow \Box \Diamond \Box \phi)$ | theorem in S5 |
| 10. | $(\Box \phi \rightarrow \Box \Box \phi)$ | chain rule, 9, 8 |

$$((\Box A \wedge \Diamond B) \rightarrow \Diamond(A \wedge B))$$

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|-----|--|-------------------|
| 1. | $(\Box A \wedge \Diamond B)$ | ACD/hypothesis |
| 2. | $\Box A$ | simplification, 1 |
| 3. | $\Diamond B$ | simplification, 1 |
| 4. | $\neg \Diamond(A \wedge B)$ | AID |
| 5. | $\neg \neg \Box \neg(A \wedge B)$ | definition, 4 |
| 6. | $\Box \neg(A \wedge B)$ | DN, 4 |
| 7. | $\Box \neg \neg (A \rightarrow \neg B)$ | equivalence, 6 |
| 8. | $\Box (A \rightarrow \neg B)$ | DN, 7 |
| 9. | $(\Box (A \rightarrow \neg B) \rightarrow (\Box A \rightarrow \Box \neg B))$ | M2 |
| 10. | $(\Box A \rightarrow \Box \neg B)$ | MP 8, 9 |
| 11. | $\Box \neg B$ | MP2, 10 |
| 12. | $\neg \Diamond \neg \neg B$ | equivalence, 11 |
| 13. | $\neg \Diamond B$ | DN12 |
| 14. | $\Diamond B$ | repeat 3 |
| 15. | $\Diamond(A \wedge B)$ | ID 4-14 |
| 16. | $((\Box A \wedge \Diamond B) \rightarrow \Diamond(A \wedge B))$ | CD 1-15 |

$$\Box A \vdash\!\!\vdash \Box(B \rightarrow A)$$

1. $(A \rightarrow (B \rightarrow A))$ L2 (or: theorem)
2. $\Box(A \rightarrow (B \rightarrow A))$ nec 1
3. $(\Box(A \rightarrow (B \rightarrow A)) \rightarrow (\Box A \rightarrow \Box(B \rightarrow A)))$ m2
4. $(\Box A \rightarrow \Box(B \rightarrow A))$ MP 2, 3
5. $\Box A$ premise
6. $\Box(B \rightarrow A)$ MP 4, 5

Shakey's program verified

1.	OT	premise
2.	$O\neg M$	premise
3.	$(T \rightarrow OG)$	premise
4.	$(\neg M \rightarrow OH)$	premise
5.	$(OT \rightarrow ((T \rightarrow OG) \rightarrow OG))$	D_B4
6.	$(O\neg M \rightarrow ((\neg M \rightarrow OH) \rightarrow OH))$	D_B4
7.	$((T \rightarrow OG) \rightarrow OG)$	MP1, 5
8.	$((\neg M \rightarrow OH) \rightarrow OH)$	MP2, 6
9.	OG	MP 3, 7
10.	OH	MP 4, 8
11.	$(O(G \wedge H) \leftrightarrow (OG \wedge OH))$	D_B2
12.	$(OG \wedge OH)$	adj, 9, 10
13.	$O(G \wedge H)$	EQ, 11, 12